

Indian Statistical Institute
Class Test-II
B.Math I Year
Probability Theory I
Date:25-10-04

1. Let $0 < p < 1$ and $n \geq 1$. Let m be the unique integer such that $(n+1)p - 1 < m \leq (n+1)p$. Let $a(n) = \binom{n}{m} \left(\frac{m}{n}\right)^n \left(1 - \frac{m}{n}\right)^{n-m}$. Show that $\lim_{n \rightarrow \infty} a(n) \sqrt{2\pi m \left(1 - \frac{m}{n}\right)} = 1$. (20)
2. How many independent bridge dealings (of 13 cards) are required in order for the probability of a pre-assigned player having four aces atleast once to be $1/2$ or better? Solve again for 'any one of four players' instead of a given one. (10)
3. In a book of n pages, the rate of occurrence of misprints per page is ' λ '. Find the probability that atleast one page will contain more than k misprints. (10)
4. Suppose that the probability of an insect laying r eggs is $d^{-\lambda} \frac{\lambda^r}{r!}$, $\lambda > 0$. Suppose further that the probability of an egg developing is p , $0 < p < 1$. Assuming the eggs to develop independently, show that the probability of a total of k survivors is given by $e^{-\lambda p} \frac{(\lambda p)^k}{k!}$. (15)